

B.C.A. 1st Semester (Full and Re-appear)

Examination, November-2023

MATHEMATICS

Paper-BCA-103

Time allowed : 3 hours] [Maximum marks : 80

Note : Attempt five questions in all, selecting one question from each unit. Question No. 1 is compulsory. All questions carry equal marks.

1. (a) If $A = \{1, 3, 5, 7\}$, $B = \{2, 4, 6, 8\}$ and $C = \{0, 4, 5\}$. Write $A \cap B$ and $B \cup C$.

(b) If $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. Compute $A - B$.

- (c) If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} 3x - 1, & \text{if } x > 3 \\ x^2 - 2, & \text{if } -2 \leq x \leq 3 \\ 2x + 3, & \text{if } x < -2 \end{cases}$$

Find (i) $f(4)$, (ii) $f(-3)$

(d) Evaluate : $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$

(e) If $y = 5x^3 + 8x^2 - 7x + 10$, find $\frac{dy}{dx}$.

(f) If $y = \cos(2x + 1)$, find $\frac{dy}{dx}$.

(g) Evaluate $\int (2x + 1)^{\frac{2}{3}} dx$

(h) Evaluate $\int_1^2 (3x - 2) dx$

Unit-I

2. (a) If $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ and $C = \{2, 6, 7, 10\}$, then verify that

(i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

(b) For a certain test a candidate could offer English or Hindi or both the subjects. Total number of students was 500, of whom 350 appeared in English and 90 in both subjects.

(i) How many appeared in English only?

(ii) How many appeared in Hindi?

(iii) How many appeared in Hindi only?

3. (a) Prove that

$$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$

[P.T.O.]

(b) Solve the following system of equations :

$$6x + y - 3z = 5$$

$$x + 3y - 2z = 5$$

$$2x + y + 4z = 8$$

Unit-II

4. (a) Determine whether the Relation R in the set

$$A = \{4, 5, 6, 7\} \text{ defined as } R = \{(4, 5), (5, 4),$$

$(7, 6), (6, 7)\}$ is reflexive, symmetric, transitive or anti symmetric ?

(b) Prove that f is a bijective function and hence find

its inverse, f^{-1} where $f : R \rightarrow R$ is defined as

$$f(x) = 2x + 3.$$

5. (a) (i) Evaluate : $\lim_{x \rightarrow 0} \frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$

(ii) Evaluate : $\lim_{x \rightarrow 0} \frac{x}{|x|}$

(b) Find the value of a if the function f is given by

$$f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$$

is continuous at $x = 2$.

Unit-III

6. Differentiate the following w.r.t. x

(i) $(x^2 - 4x + 5)(x^3 - 2)$

(ii) $\left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2$

(iii) $\frac{3x^2 - 2}{x^2 + 7}$

(iv) $\sqrt{1 + x^2}$

7. Differentiate the following w.r.t. x

(i) $\frac{x}{1 + \tan x}$

(ii) $(\sin^{-1} x)^2$

(iii) $\cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right)$

(iv) $e^x \log x$

Unit-IV

8. (i) Evaluate: $\int \frac{1}{\sqrt{2x - x^2}} dx$

(ii) Evaluate: $\int x^2 \sin x dx$

9. (i) Evaluate : $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

(b) Evaluate : $\int_0^{\pi/2} \frac{\sin \theta}{\sqrt{1+\cos \theta}} d\theta$.